Integer Frank-Wolfe for Optimal Experiment Design

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Berlin Mathematics Research Center

Think scottish shortbread

Outline

1. A- and D-Optimal Experiment Design
2. Integer Frank-Wolfe: `Boscia.jl`
3. E-Optimal Design and Nesterov Smoothing
4. Summary

D-and A-Optimal Experiment Design

$$\begin{aligned} \min_{\mathbf{x}} \quad & -\log\det\left(A^\intercal \operatorname{diag}(\mathbf{x}) A\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} x_i = N \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \\ & \mathbf{x} \in \mathbb{N}_0^m, \end{aligned} \tag{D}$$

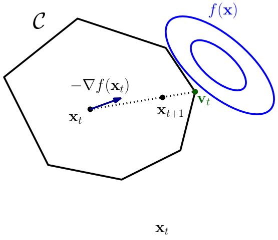
Ponte, Fampa, and Lee 2025; Ahipaşaoğlu 2021; Li et al. 2024

$$\begin{aligned} \min_{\mathbf{x}} \quad & \operatorname{tr}\left(\left(A^\intercal \operatorname{diag}(\mathbf{x}) A\right)^{-1}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} x_i = N \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \\ & \mathbf{x} \in \mathbb{N}_0^m, \end{aligned} \tag{A}$$

Ahipaşaoğlu 2015; Sagnol and Pauwels 2019; Nikolov, Singh, and Tantipongpipat 2022

where A encodes the experiments and $M(\mathbf{x}) = A^\intercal \operatorname{diag}(\mathbf{x}) A$ is referred to as the *Information matrix*.

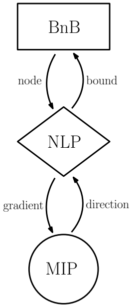
Integer Frank-Wolfe: `Boscia.jl`



Frank and Wolfe 1956; Levitin and Polyak 1966; Braun et al. 2022

$$\min_{\mathbf{x}} f(\mathbf{x})$$
$$\text{s.t. } \mathbf{x} \in \mathcal{X}$$
$$\mathbf{x}_j \in \mathbb{Z} \quad \forall j \in J$$

- f continuously differentiable, convex, L-smooth.
- $\mathcal{X}$ is a compact convex set, usually polyhedral.

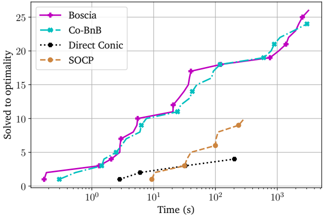


Schematic of the algorithm.

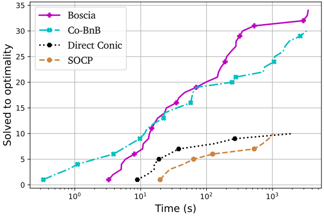
Hendrych et al. 2025

Experimental Results

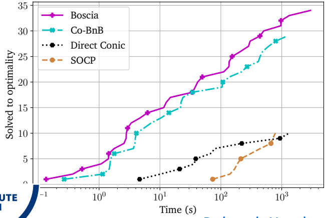
A-Optimal (Independent)



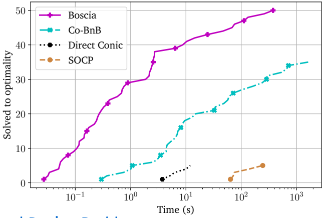
A-Optimal (Correlated)



D-Optimal (Independent)



D-Optimal (Correlated)



E-Optimal Experiment Design

$$\begin{aligned}
\max_{\mathbf{x}}\ & \lambda_{\min}\left(A^\intercal \operatorname{diag}(\mathbf{x}) A\right) \\
\text{s.t.}\ & \sum_{i=1}^{m} x_i = N \\
& \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \\
& \mathbf{x} \in \mathbb{N}_0^m,
\end{aligned} \tag{E}$$

Brown, Laddha, and Singh 2024; Rendi, Vanderbei, and Wolkowicz 1995; Allen-Zhu et al. 2021

E-Criterion and smoothing

Issue

The subgradients of $\lambda_i(X)$ are the corresponding eigenvectors.
$\Rightarrow$ the gradient $\nabla\lambda_i$ only exists if $\mathrm{mult}(\lambda_i) = 1$.

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Solution - Smoothing

We use the smoothing introduced in Nesterov 2007.

$$\max \lambda_{\min}(X) \Leftrightarrow \min \lambda_{\max}(-X)$$

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$$\max \lambda_{\min}(X) \Leftrightarrow \min \lambda_{\max}(-X)$$

$$f_\mu(X) = -\mu \log\left(\sum_{i=1}^{n} e^{-\lambda_i(X)/\mu}\right) + \mu \log n \tag{1}$$

$$f_\mu(X) \geq f(X) \geq f_\mu(X) + \mu \log n \quad \forall X \in \mathbb{S}^n \tag{2}$$

Pruning, Dual Tightening and Approximate Gradient

Pruning

- Prune in the case of rank deficiency.
- Prune based on the maximal achievable eigenvalue.

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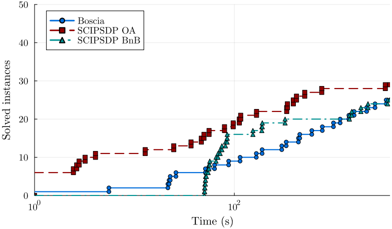
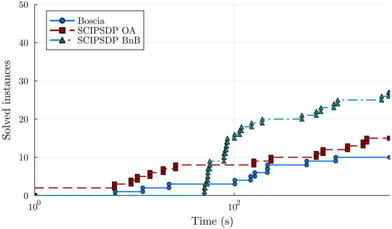
Approximate Gradient

- Drop the largest eigenvalues from the gradient inspired by d'Aspremont 2008.

Computational Results



correlated data



independent data

Another Application - Maximum Algebraic Connectivity

$$\begin{aligned} &\max \lambda_2(L_{G(\mathbf{x})}) \\ &\text{s.t. } G(\mathbf{x}) \text{ satisfies graph some conditions} \end{aligned} \tag{3}$$

where $L_{G(\mathbf{x})}$ is the Laplacian of a subgraph of an undirected (weighted) graph G induced by $\mathbf{x}$.

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By Chan 2018 and Lamperski, Yang, and Prokopyev 2024, we have

$$\lambda_2(L) = \lambda_{\min}(L + \mathbb{1}\mathbb{1}^T)$$

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Somisetty, Nagarajan, and Darbha 2024 introduce a Maximum Algebraic Connectivity problem over spanning trees of a weighted graph.

Computational Results

Table: Performance comparison for ACST problem grouped by dimension. Geometric mean of the time is computed with a 1s shift. There are 12 instances in total.

Solver	Metric	45	66	105	300	all
Boscia	% sol.	**100.0**	0.0	0.0	0.0	**14.3**
	time (s)	**500.8**	3600.0	3600.3	3602.4	**2716.4**
	rel. gap	—	**6.01e-01**	**2.01e+00**	**5.97e+00**	**6.84e+00**
SCIPSDP (OA)	% sol.	0.0	0.0	0.0	0.0	0.0
	time (s)	3600.0	3600.0	3600.0	3600.0	3600.2
	rel. gap	—	—	5.53e+00	1.00e+20	7.47e+15
SCIPSDP (B&B)	% sol.	0.0	0.0	0.0	0.0	0.0
	time (s)	3600.0	3600.0	3600.0	3600.0	3600.0
	rel. gap	1.08e-01	7.24e-01	3.13e+00	2.09e+01	2.23e+04

Summary

- B&B with Frank-Wolfe scales better when the SOCP and SDP formulations need to add a lot of additional variables and/or constraints

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- In case of harder combinatorial and/or linear constraints, decoupling them from the PSD constraints via smoothing shows the superior performance.

Summary

- B&B with Frank-Wolfe scales better when the SOCP and SDP formulations need to add a lot of additional variables and/or constraints
- The SDP formulation for maximizing the minimum eigenvalue under simple budget constraints is tight by the theory and the computational results support that.
- In case of harder combinatorial and/or linear constraints, decoupling them from the PSD constraints via smoothing shows the superior performance.
- Can we find tighter first-order approximations?

Thank you for your attention!

Convex mixed-integer optimization with Frank–Wolfe methods

Solving the Optimal Experiment Design Problem with Mixed-Integer Convex Methods

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References I

Ahipaşaoğlu, Selin Damla (2015). "A first-order algorithm for the A-optimal experimental design problem: a mathematical programming approach". In: *Statistics and Computing* 25.6, pp. 1113–1127.

— (2021). "A branch-and-bound algorithm for the exact optimal experimental design problem". In: *Statistics and Computing* 31.5, p. 65.

Allen-Zhu, Zeyuan et al. (2021). "Near-optimal discrete optimization for experimental design: A regret minimization approach". In: *Mathematical Programming* 186.1, pp. 439–478.

Braun, Gábor et al. (2022). "Conditional gradient methods". In: *arXiv preprint arXiv:2211.14103*.

Brown, Adam, Aditi Laddha, and Mohit Singh (2024). "Fast algorithms for maximizing the minimum eigenvalue in fixed dimension". In: *Operations Research Letters* 57, p. 107186.

Carderera, Alejandro, Mathieu Besançon, and Sebastian Pokutta (2021). "Simple steps are all you need: Frank-Wolfe and generalized self-concordant functions". In: *Advances in Neural Information Processing Systems* 34, pp. 5390–5401.

Chan, Pak Hay (2018). "The Complexity of Network Design for st Effective Resistance". Master's thesis. Waterloo, Ontario, Canada: University of Waterloo.

Coey, Chris, Lea Kapelevich, and Juan Pablo Vielma (2022a). "Performance enhancements for a generic conic interior point algorithm". In: *Mathematical Programming Computation*. DOI: https://doi.org/10.1007/s12532-022-00226-0.

— (2022b). "Solving natural conic formulations with Hypatia.jl". In: *INFORMS Journal on Computing* 34.5, pp. 2686–2699. DOI: https://doi.org/10.1287/ijoc.2022.1202.

References II

Coey, Chris, Miles Lubin, and Juan Pablo Vielma (2020). "Outer approximation with conic certificates for mixed-integer convex problems". In: *Mathematical Programming Computation* 12.2, pp. 249–293.

d'Aspremont, Alexandre (2008). "Smooth optimization with approximate gradient". In: *SIAM Journal on Optimization* 19.3, pp. 1171–1183.

Frank, Marguerite and Philip Wolfe (1956). "An algorithm for quadratic programming". In: *Naval research logistics quarterly* 3.1-2, pp. 95–110.

Hendrych, Deborah et al. (2025). "Convex mixed-integer optimization with Frank-Wolfe methods". In: *Mathematical Programming Computation*.

Lamperski, Jourdain, Haeseong Yang, and Oleg Prokopyev (2024). "Simple Randomized Rounding for Max-Min Eigenvalue Augmentation". In: *Forty-second International Conference on Machine Learning*.

Levitin, Evgeny S and Boris T Polyak (1966). "Constrained minimization methods". In: *USSR Computational mathematics and mathematical physics* 6.5, pp. 1–50.

Li, Yongchun et al. (2024). "D-Optimal Data Fusion:: Exact and Approximation Algorithms". In: *INFORMS Journal on Computing* 36.1, pp. 97–120.

Nesterov, Yurii (2007). "Smoothing technique and its applications in semidefinite optimization". In: *Mathematical Programming* 110.2, pp. 245–259.

Nikolov, Aleksandar, Mohit Singh, and Uthaipon Tantipongpipat (2022). "Proportional volume sampling and approximation algorithms for A-optimal design". In: *Mathematics of Operations Research* 47.2, pp. 847–877.

References III

Ponte, Gabriel, Marcia Fampa, and Jon Lee (2025). "Branch-and-bound for D-Optimality with fast local search and variable-bound tightening". In: *Mathematical Programming*, pp. 1–38.

Rendi, Franz, Robert J Vanderbei, and Henry Wolkowicz (1995). "Max-min eigenvalue problems, primal-dual interior point algorithms, and trust region subproblemst". In: *Optimization Methods and Software* 5.1, pp. 1–16.

Sagnol, Guillaume (2011). "Computing optimal designs of multiresponse experiments reduces to second-order cone programming". In: *Journal of Statistical Planning and Inference* 141.5, pp. 1684–1708.

Sagnol, Guillaume and Radoslav Harman (2015). "Computing exact D-optimal designs by mixed integer second-order cone programming". In: *The Annals of Statistics* 43.5, pp. 2198–2224.

Sagnol, Guillaume and Edouard Pauwels (2019). "An unexpected connection between Bayes A-optimal designs and the group lasso". In: *Statistical Papers* 60.2, pp. 565–584.

Somisetty, Neelkamal, Harsha Nagarajan, and Swaroop Darbha (2024). "Optimal robust network design: Formulations and algorithms for maximizing algebraic connectivity". In: *IEEE Transactions on Control of Network Systems* 12.1, pp. 918–929.

Zhao, Renbo (2025). "New Analysis of an Away-Step Frank-Wolfe Method for Minimizing Log-Homogeneous Barriers". In: *Mathematics of Operations Research*.

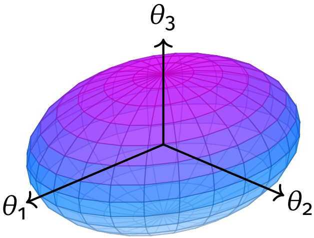
Set-up

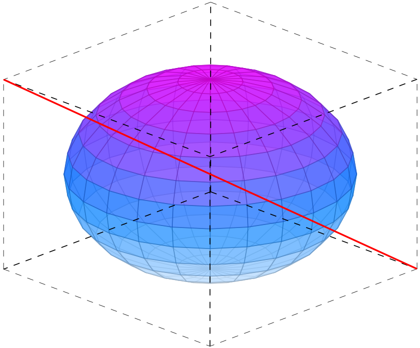
- The ultimate goal is fitting a regression model.

$$\min_{\boldsymbol{\theta} \in \mathbb{R}^n} \|A\theta - \mathbf{y}\|_2^2$$

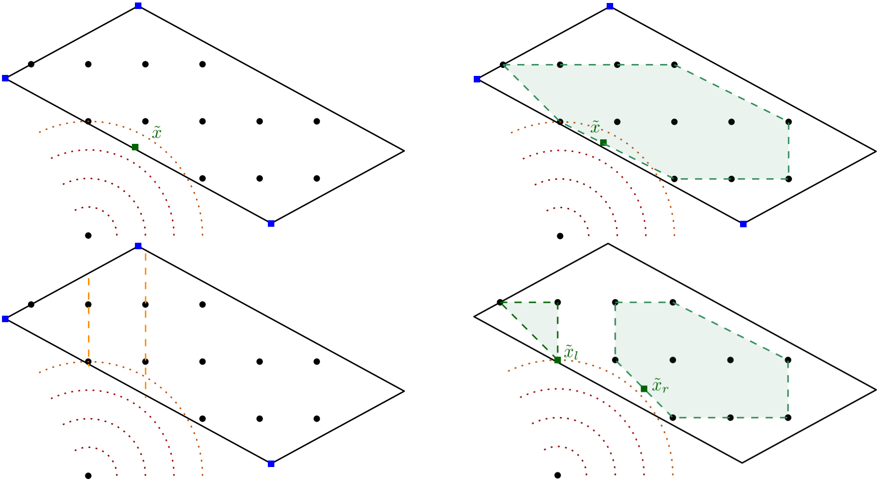
- $A = [-\mathbf{a}_i-]_{i=1}^m$ with $\mathbf{a}_i \in \mathbb{R}^n$ is the *Experiment Matrix*. Assumed to have full column rank.
- θ is the unknown set of parameters to be found.
- $\mathbf{y}$ is the **not-yet** measured response.
- Issue: Running all m experiments too costly and/or too time intensive.

Graphical interpretation

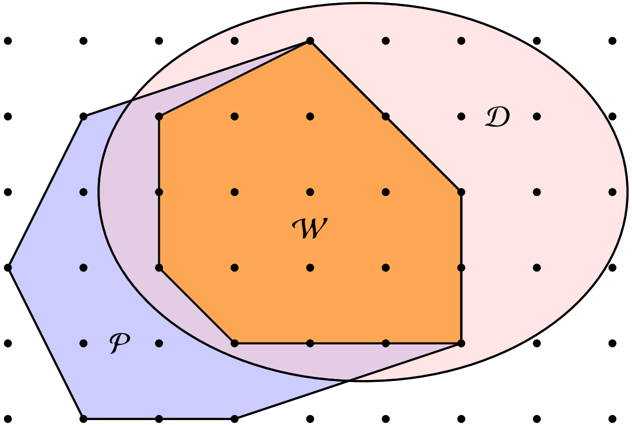




Convex relaxation vs integer hull



Feasible region vs Domain



A schematic representation of the feasible region $\mathcal{P}$, the domain of the objective $\mathcal{D}$ and the convex hull of vertices that are both feasible and in the domain denoted as $\mathcal{W}$.

Generalized self-concordance

By Carderera, Besançon, and Pokutta 2021, we also have convergence if the objective is generalized self-concordant. Self-concordance was already proved for the $-\log\det(X)$ on $\mathbb{S}^n_{++}$.

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Theorem (GSC A-criterion hendrych2024optimal)

The function $g(X) = \operatorname{tr}(X^{-p})$, with $p > 0$, is $\left(3, \frac{(p+2)\sqrt[4]{a^{2p}n}}{\sqrt{p(p+1)}}\right)$-generalized self-concordant on $\mathrm{dom}(g) = \left\{X \in \mathbb{S}^n_{++} \mid 0 \prec X \preccurlyeq aI\right\}$ where $a \in \mathbb{R}_{>0}$ bounds the maximum eigenvalue of X.

Other approaches

Co-BnB Ahipaşaoğlu 2021

- Branch-and-Bound with customized coordinate-descent-like algorithm for the node problems, exploits connection to Minimum Volume Enclosing Ellipsoid Problem.
- Developed for $m >> n$.

Outer Approximation approaches

- Using the mixed-integer convex solver with conic certificates `Pajarito.jl` Coey, Kapelevich, and Vielma 2022a.
    1. **Direct Conic:** Natural formulation via `Hypatia.jl` Coey, Kapelevich, and Vielma 2022b.
    2. **SOCP:** as introduced in Sagnol 2011; Sagnol and Harman 2015.

Convergence results

Objective	Convergence	Linear Convergence
$f(\mathbf{x}) = -\log\det(X(\mathbf{x}))$	✓	✓*
$g(\mathbf{x}) = \mathrm{tr}(X(\mathbf{x})^{-p})$	✓	Not guaranteed
$k(\mathbf{x}) = \log\left(\mathrm{tr}(X(\mathbf{x})^{-p})\right)$	Conjecture	Not guaranteed

Table: Convergence of Frank-Wolfe on different objective functions.

*We adapted the proof from Zhao 2025 to the Blended Pairwise Conditional Gradient (BPCG).

Hendrych, D. and Besançon, M. and Pokutta, S. (2025). "Solving the Optimal Experiment Design Problem with Mixed-Integer Convex Methods." https://arxiv.org/abs/2312.11200

What do we need for convergence?

In short: Frank-Wolfe has to converge.

$$f(X) = -\log\det(X) \quad \text{and} \quad g(X) = \operatorname{tr}(X^{-p}) \quad \text{and} \quad k(X) = \log\operatorname{tr}(X^{-p})$$

We need:

- Convexity of the objective functions. ✓
- L-smoothness of the objective functions. → Issue with the domain.
- Generalized self-concordance of the objective functions.
- Strong convexity/Sharpness of the objective functions.

Hendrych, D. and Besançon, M. and Pokutta, S. (2025). "Solving the Optimal Experiment Design Problem with Mixed-integer Convex Methods." `https://arxiv.org/abs/2312.11200`

L-smoothness

Theorem (L-smoothness for the Fusion Problems)

The functions $f_F(\mathbf{x}) = \log\det(X_C(\mathbf{x}))$, $g_F(\mathbf{x}) = \mathrm{tr}(X_C(\mathbf{x})^{-p})$ *and* $k_F(\mathbf{x}) = \log\left(\mathrm{tr}(X_C(\mathbf{x})^{-p})\right)$ *are L-smooth on* $x \in \mathbb{R}_{\geq 0}$*.*

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Theorem (Local L-smoothness for the Optimal Problems)

The functions $f(X(\mathbf{x}))$, $g(X(\mathbf{x}))$ and $k(X(\mathbf{x}))$ are locally L-smooth on

$$\mathcal{L}_0 = \{\mathbf{x} \in \mathcal{D} \cap \mathcal{P} \mid (*)(\mathbf{x}) \leq (*)(\mathbf{x}_0)\}$$

where $(*)$ is a placeholder for each function and $x_0 \in \mathcal{D}$ an initial point.

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Sharpness

Theorem (Strong convexity)

The functions $f(X) = -\log\det(X)$ *and* $g(X) = \operatorname{tr}(X^{-p})$, $p > 0$, *are strongly convex on* $D := \{X \in \mathbb{S}^n_{++} \mid \lambda_{\max}(X) \leq \alpha\}$.
The function $k(X) = \log\operatorname{tr}(X^{-p})$, $p > 0$, *is strongly convex on* $D := \{X \in \mathbb{S}^n_{++} \mid \lambda_{\max}(X) \leq \alpha, \kappa(X) \leq \kappa\}$.

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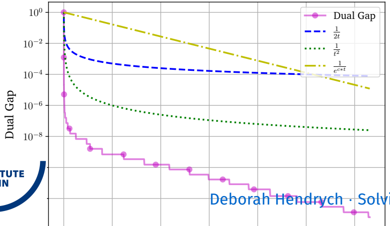
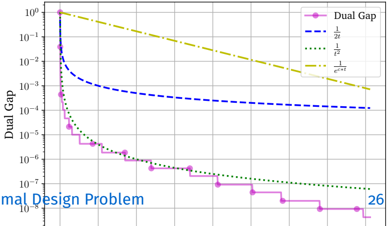
Theorem

The compositions of f, g *and* k *with the information matrix* $X(\mathbf{x})$, *respectivily, are sharp on* $\mathcal{W}$.

In general, the compositions do not have a unique minimizer.





A Custom Branch-and-Bound for OEDP under Matrix Means (Co-BnB)

Ahipaşaoğlu 2021

- Coordinate-Descent-like algorithm for the nodes.
- Developed for $m \gg n$.

$$\begin{aligned} \max_{\mathbf{w}} \ & \log(\phi(X(\mathbf{w}))) \\ \text{s.t.} \ & \sum_{i=1}^{m} w_i = 1 \\ & \mathbf{w} \in [0,1]^m \\ & N\mathbf{w} \in \mathbb{Z}^m_{\geq 0} \end{aligned} \tag{M-OEDP}$$

- **w** can be interpreted as a probability distribution.
- Exploit connection to the Minimum Volume Enclosing Ellipsoid Problem (MVEP) for the termination criteria.

Direct Conic Formulation

coey2022conic; Coey, Kapelevich, and Vielma 2022a; Coey, Lubin, and Vielma 2020

- `Pajarito.jl` is a mixed-integer convex solver with conic certificates.
- `Hypatia.jl` is an interior point solver for conic optimization problems.
- D-Criterion:

$$\mathcal{K}_{\log\det} := \operatorname{cl}\left\{(u, v, W) \in \mathbb{R} \times \mathbb{R}_{>0} \times \mathbb{S}^n_{++} \mid u \leq v \log\det(W/v)\right\}$$

- A-Criterion: Dual of

$$\mathcal{K}_{\text{sepspec}} := \operatorname{cl}\left\{(u, v, w) \in \mathbb{R} \times \mathbb{R}_{>0} \times \operatorname{int}(\mathcal{Q}) \mid u \geq v\varphi(w/v)\right\}$$

- $\mathcal{Q}$ is the PSD cone and φ is the negative square root.
- The convex conjugate of the negative square root is the trace inverse.

Second-Order Conic Formulation (SOCP)

Sagnol 2011; Sagnol and Harman 2015

- In Sagnol 2011 the SOCP formulation for the continuous problem was introduced.
- The SOCP formulation of the exact formulation of OEDP was shown in Sagnol and Harman 2015.
- In theory, a nice result, but in practice, the problem size becomes much larger.
- For the A-Optimal Problem, we have $2m(n+1)$ variables and $2(n+1)+m$ constraints.
- For the D-Optimal Problem, we have $2m(1+n)+n^2+1$ variables and $n(m+1)+3m+4$ constraints.

SOCP A-Criterion

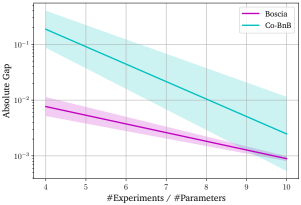
$$\min_{\substack{\boldsymbol{\mu}\in\mathbb{R}^m \\ \mathbf{x}\in\mathbb{Z}^m \\ \forall i\in[m]\ Y_i\in\mathbb{R}^{1xn}}} \quad \sum_{i=1}^m t_i$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i=1}^m A_i Z_i = I \\ & \forall i \in [m],\ \|Z_i\|_F^2 \leq t_i x_i \\ & \sum_{i=1}^m x_i = N \end{aligned} \qquad \text{(A-SOCP)}$$

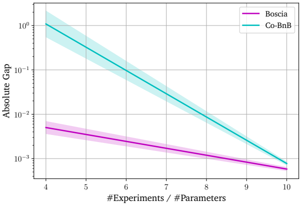
SOCP D-Criterion

$$
\begin{aligned}
\min_{\substack{\mathbf{w}\in\mathbb{R}^m_{\geq 0},\, \mathbf{x}\in\mathbb{Z}^m \\ J\in\mathbb{R}^{nxn} \\ \forall i\in[m]\ Z_i\in\mathbb{R}^{1xn} \\ \forall i\in[m], j\in[n]\ t_{ij}\in\mathbb{R}_{\geq 0}}} \quad & \prod_{i=1}^{m} J_{ii}^{1/m} \\
\text{s.t.} \quad & \sum_{i=1}^{m} A_i Z_i = J \\
& J \text{ lower triangle matrix} \\
& \forall i \in [m], j \in [n]\ \|Z_i e_j\|_F^2 \leq t_{ij} w_i \\
& \forall j \in [n]\ \sum_{i=1}^{m} t_{ij} \leq J_{jj} \\
& \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \text{ and } \sum_{i=1}^{m} w_i = 1
\end{aligned}
\tag{D-SOCP}
$$

Effect of the conditioning of the problem



A IND



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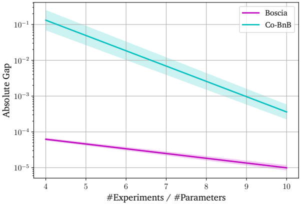
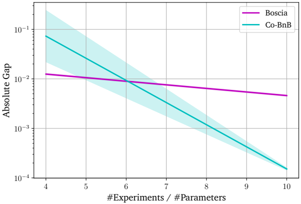


A CORR



D CORR